

23 August 2022

Arithmetic Sequences

Examples :

$d=2$ 1, 3, 5, 7, 9, 11, ...
 $d=5$ 5, 10, 15, 20, 25, 30, ...
 $d=-3$ 10, 7, 4, 1, -2, -5, ...

} Common difference = d

$$a_n = a_1 + (n-1)d$$

First series $a_1=1$ common difference $d=2$

$$a_1 = 1 + (1-1) \times 2 = 1$$

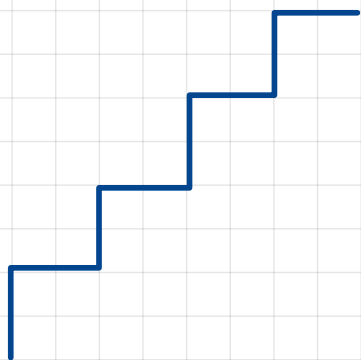
$$a_2 = 1 + (2-1) \times 2 = 3$$

$$a_n = 1 + (n-1) \times 2 = 1 + 2n - 2 = 2n - 1$$

Q1 write down a_n for the other two series

$$a_n = 5 + (n-1) \times 5 = 5 + 5n - 5 = 5n$$

$$a_n = 10 + (n-1) \times (-3) = 10 - 3n + 3 = 13 - 3n$$



stairs

Internal Assessment (IA)

Maths exploration

Arithmetic sequence

a = first term

d = common difference

The first 5 terms

$$a, a+d, a+2d, a+3d, a+4d$$

The sum of arithmetic series, s terms:

$$S_5 = a + a+d + a+2d + a+3d + a+4d$$

$$S_5 = a+4d + a+3d + a+2d + a+d + a$$

$$2S_5 = 2a+4d + 2a+4d + 2a+4d + 2a+4d + 2a+4d$$

$$2S_5 = 5(2a+4d)$$

$$S_5 = \frac{5}{2}(2a+4d)$$

$$S_3 = a + a+d + a+2d$$

$$S_3 = a+2d + a+d + a$$

$$2S_3 = 2a+2d + 2a+2d + 2a+2d$$

$$2S_3 = 3(2a+2d)$$

$$S_3 = \frac{3}{2}(2a+2d)$$

$$S_n = a + a+d + a+2d \dots a+(n-2)d + a+(n-1)d$$

$$S_n = a+(n-1)d + a+(n-2)d \dots a+d + a$$

$$2S_n = 2a+(n-1)d + \dots + 2a+(n-1)d$$

$$2S_n = n(2a+(n-1)d)$$

$$S_n = \frac{n}{2}(2a+(n-1)d)$$

knowing the proofs enhances
your understanding, paper 3
long question

Q0

$$3, 7, 11$$

$$\begin{aligned} 7-3 &= 4 \\ 11-7 &= 4 \end{aligned} \Rightarrow d=4$$

$$S_{100} = \frac{100}{2} (2 \times 3 + (100-1) \times 4) = 20100$$

Q1

$$37 = a + 6d$$

substitute for d

$$\underline{17 = a + 3d}$$

$$17 = a + 3 \times \frac{20}{3}$$

$$20 = 3d$$

$$a = -3$$

$$d = 20/3$$

Q2

$$0 = a + a + d$$

$$0 = 2a + d \quad (1)$$

$$d = -2a \text{ sub into (2)}$$

$$85 = a + 9d \quad (2)$$

$$85 = a - 18a$$

$$a = -5$$

$$d = 10$$

$$-17a = 85$$

Question 0

The first three terms of an arithmetic series are 3, 7, 11. Determine the sum of the first 100 terms.

(3 marks)

Question 1

The first term of an arithmetic series is a and the common difference is d . The seventh term is equal to 37 and the fourth term is equal to 17. Determine the value of a and d . Find the sum of the first 20 terms.

(5 marks)

Question 2

The sum of the first two terms of an arithmetic series is zero. The tenth term is equal to 85. Find the value of the first term a and the common difference d .

(5 marks)

Question 3

Find the sum of the first 1000 positive integers except integers which are multiple of 7.

(6 marks)

Question 4

The sum of the first 10 terms of an arithmetic series is x , and the sum of the first 20 terms is y . Find the first term and the common difference.

Question 5

A pipe with length 174 cm is cut in slices with increasing lengths and where the slice lengths form an arithmetic series. The first slice to be cut has a length of 7 cm and the slice to be cut has a length of 11 cm. How many slices are there?

(6 marks)

Question 6

The ratio of the second term and first term of an arithmetic series is 3. The ratio of the 10th term to the 5th term is $\frac{19}{9}$. Find the sum of the first 20 terms.

(6 marks)

Question 7

The first three terms of an arithmetic series are x , $x+1$, and x^2 . Determine the possible values of x .

(6 marks)

Question 8

The first three terms of an arithmetic series are x , $x+3$, and x^3 . Determine the value of the first term and the common difference.

(use a graphical calculator or desmos for the final step).