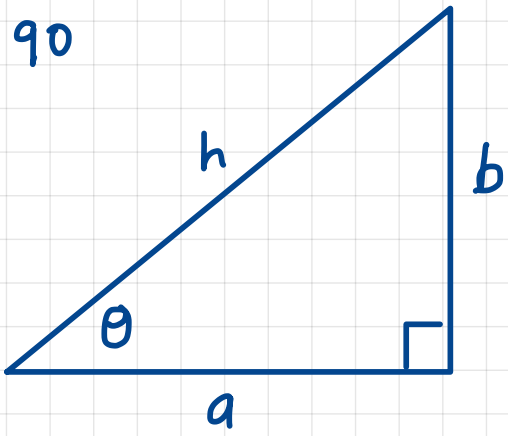
18 October 2022

$\cos\theta = \frac{a}{h}$

$0 < \theta < 90$



range of $\cos\theta$

$\theta \to 0 \Rightarrow b \to 0$

$a \to h \quad \cos\theta \to 1$

$\theta \to 90° \quad a \to 0 \quad b \to h$

$0 < \cos\theta < 1$

$\cos\theta \to 0$

Q1

$\sin\theta = \frac{b}{h}$

Take the same limits as above

$\theta \to 0 \quad \sin\theta \to 0 \quad \sin\theta \quad b \to 0$

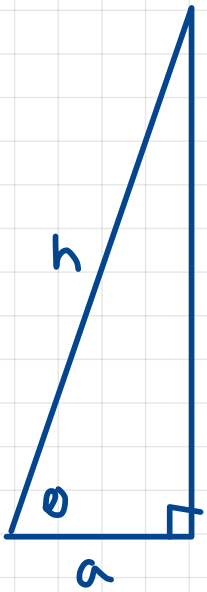
$\theta \to 90° \quad \sin\theta \to 1 \quad \sin\theta \quad b \to h$

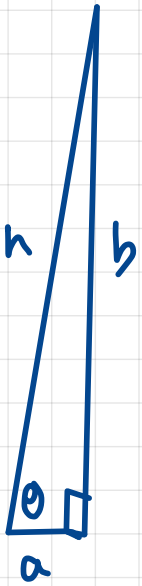
$0 < \sin\theta < 1$

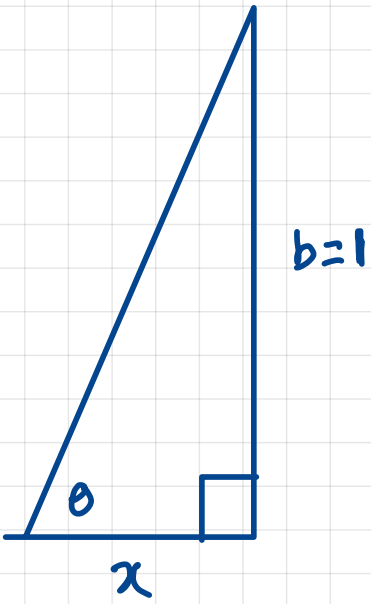
$\tan\theta \quad \theta \to 0 \quad b \to 0 \quad \tan\theta \to 0$

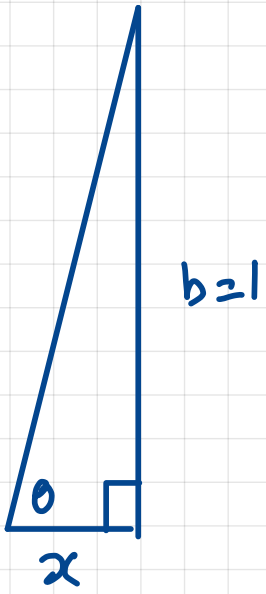
$\theta \to 90° \quad b \to h$

$a \to 0$









$\tan\theta = \frac{1}{x}$

$x \to 0 \quad \theta \to 90$

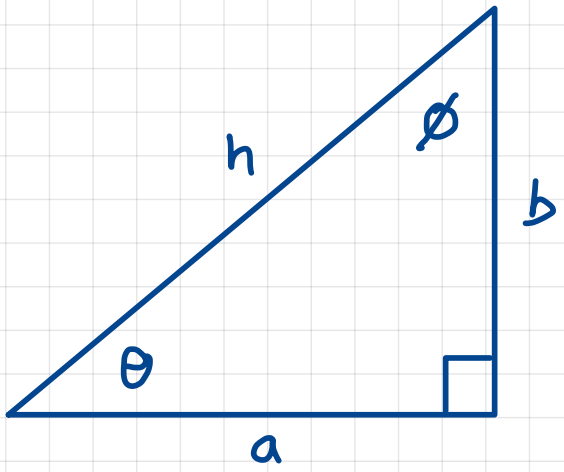
$\tan\theta \to \infty$

x	$\tan\theta$
100	0.01
10	0.1
1	1
0.1	10
0.01	100
0.001	1000
0.0001	10000
0.000001	

$0 < \theta < 90$

$0 < \tan\theta < \infty$

Q2



(i) write down the trig ratios for θ and ϕ.

(ii) what is ϕ in terms of θ

(iii) using (i) determine some trig identities

Solution

(ii) $\sin\theta = \frac{b}{h}$ $\sin\phi = \frac{a}{h}$

$\cos\theta = \frac{a}{h}$ $\cos\phi = \frac{b}{h}$

$\tan\theta = \frac{b}{a}$ $\tan\phi = \frac{a}{b}$

(ii) $\theta + \phi + 90 = 180$

$\phi = 90 - \theta$

$\sin\theta \equiv \cos(90-\theta)$

$\cos\theta \equiv \sin(90-\theta)$

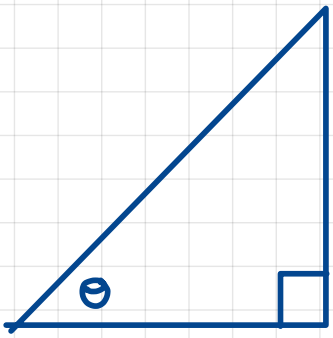
$\tan\theta \equiv \frac{1}{\tan(\theta-90)}$

use your calculator to convince your self this is true.

$$\frac{\sin\theta}{\cos\theta} = \frac{b}{h} \div \frac{a}{h} = \frac{b}{h} \times \frac{h}{a} = \frac{b}{a} = \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$$

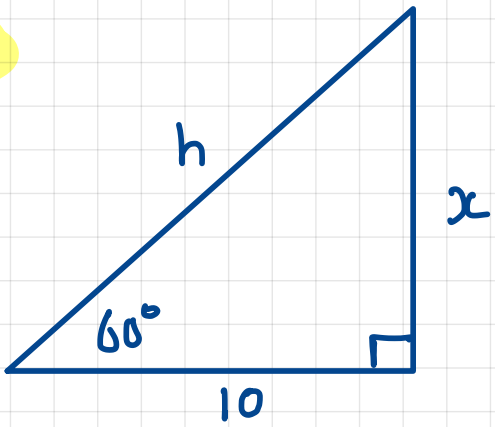
Applications of Trigonometry



given θ and one side, find the other sides

find the other sides

Q1



Solution

$$\cos 60 = \frac{10}{h}$$

$$h \cos 60 = 10$$

$$h = \frac{10}{\cos 60} = \frac{10}{1/2} = 20$$

find x use pymagoras's theorem

$$20^2 = 10^2 + x^2$$

$$400 - 100 = x^2$$

$$x = \sqrt{300} = \sqrt{3 \times 100}$$

$$= \sqrt{3}\sqrt{100} = 10\sqrt{3}$$

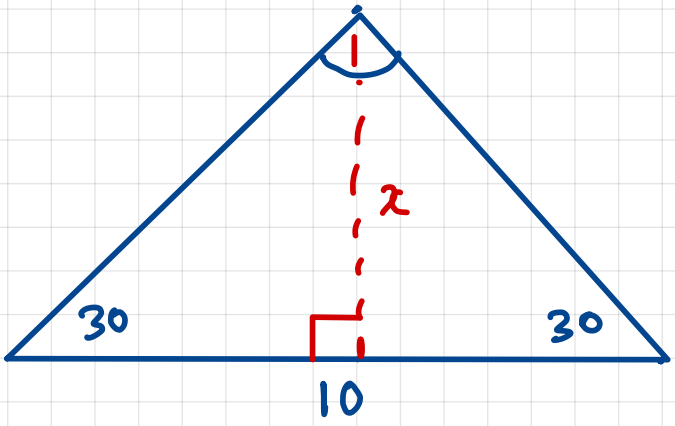
$$\tan 60 = \frac{x}{10}$$

$$x = 10 \tan 60$$

$x = 10\sqrt{3}$ since $\tan 60 = \sqrt{3}$

Q2

Find the area of the triangle



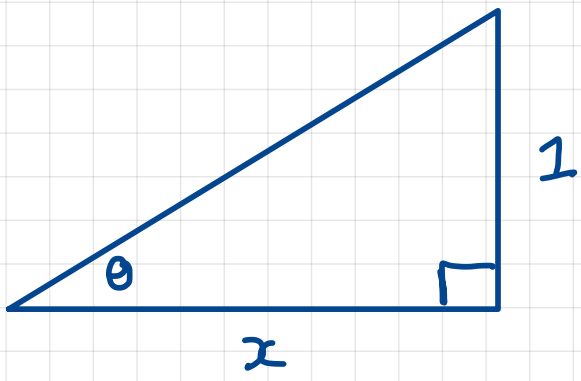
$\tan 30 = \frac{x}{5}$

$x = 5 \tan 30 = \frac{5}{\sqrt{3}}$

$\text{Area} = \frac{1}{2} \times 10 \times \frac{5}{\sqrt{3}}$

$= \frac{25}{\sqrt{3}}$

Q3 Given two sides find θ



x	θ	$1/x = q$
10	5.7	0.1
1	45	1
0.1	84.28	10
0.001	89.94	1000
0.0001	89.99	10000

$$\tan\theta = \frac{1}{x}$$

$$\theta = \arctan\left(\frac{1}{x}\right)$$ sometimes $\theta = \tan^{-1}(1/x)$

This is a calculator function

$\arctan(q)$ if $\tan\theta = q$

what is θ