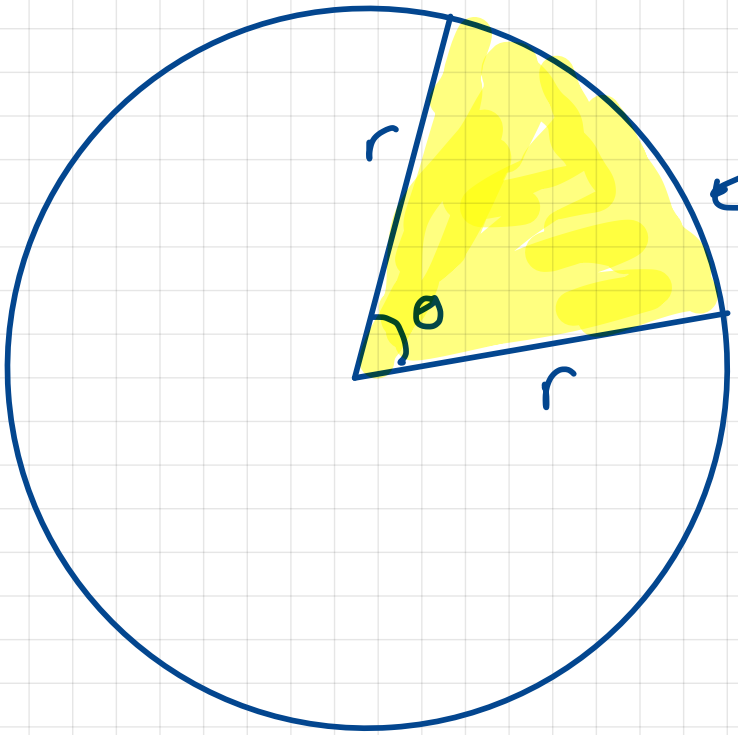
28 October 2022

conversion from degrees to radians

$$\theta_d = \frac{180}{\pi}\theta_r$$



Q1 Derive a formula for the sector area A where θ is in radians

step 1: Area of circle $= \pi r^2$

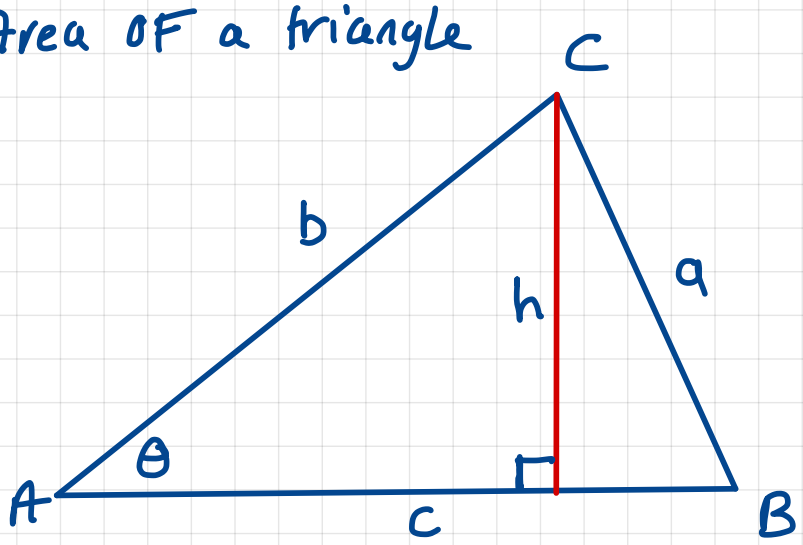
$$A = \frac{\theta_d}{360}\pi r^2$$

$$A = \frac{\pi}{360}\,\frac{180}{\pi}\,\theta_r r^2$$

$$A = \frac{\theta r^2}{2}$$

Area OF a triangle

Area $= \frac{1}{2}$ base x height



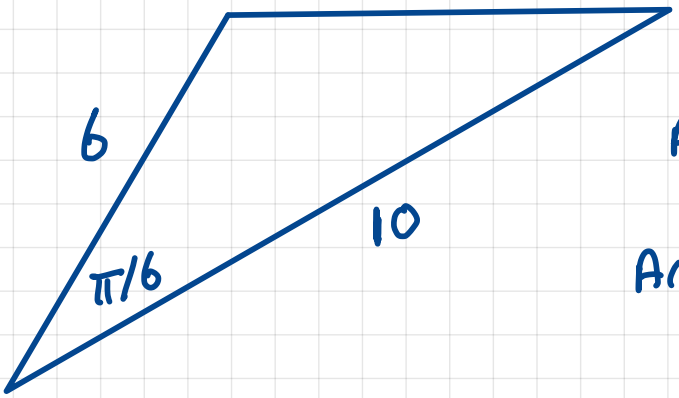
Area $= \frac{1}{2} ch$

Find an expression for h in terms of θ

$$\sin\theta = \frac{h}{b} \Rightarrow \quad h = b\sin\theta$$

Area $= \frac{1}{2} bc\sin\theta$

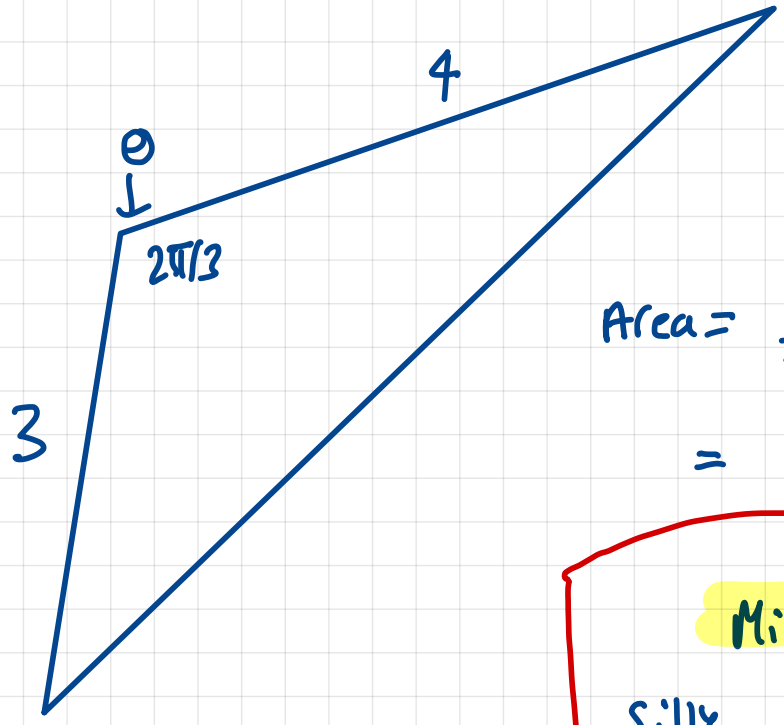
Example



Area $= \frac{1}{2} \times 6 \times 10 \sin\frac{\pi}{6}$

Area $= 30 \sin\frac{\pi}{6}$

$= 15$



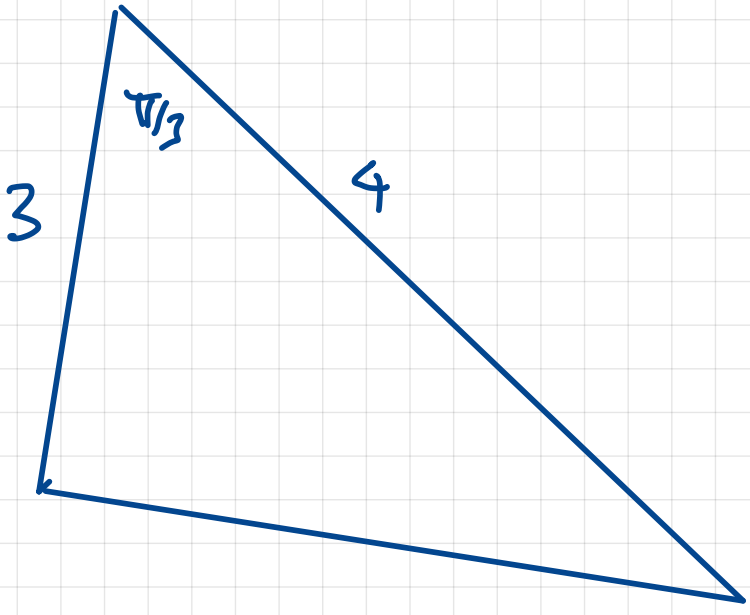
$$\text{Area} = \frac{1}{2} \times 3 \times 4 \sin\left(\frac{2\pi}{3}\right)$$

$$= 6 \sin\left(\frac{2\pi}{3}\right) = 6 \times \frac{\sqrt{3}}{2}$$

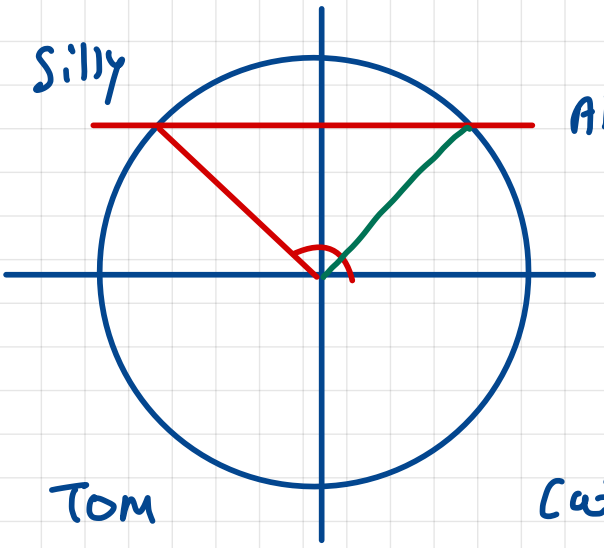
$$= 3\sqrt{3}$$

Q1 Change θ in above triangle so that the area is the same

answer



Mind

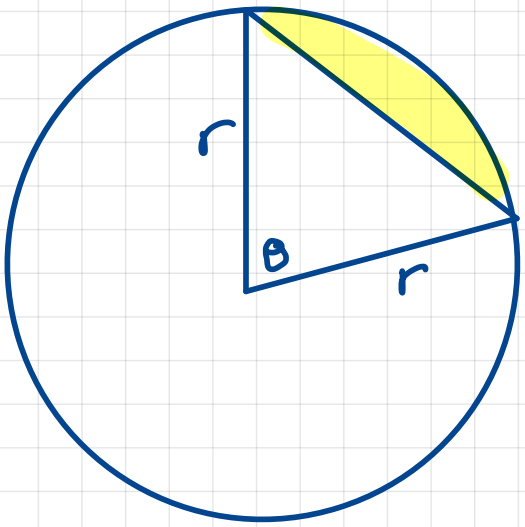


$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2}$$

Question 3

$0 < \theta < \pi$



derive an expression for the shaded region

(i) what is the minimum area of the shaded region

(ii) Find the maximum possible area (with and without technology)

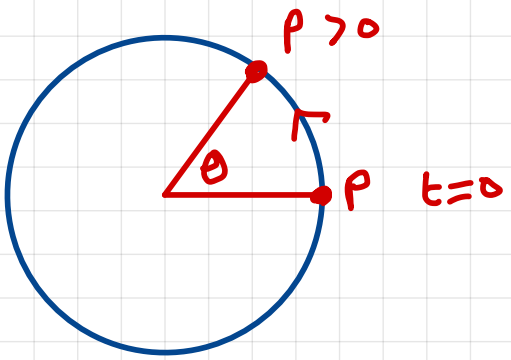
$$A(\theta) = \frac{\theta}{2} r^2 - \frac{1}{2} r^2 \sin\theta$$

$$A(\theta) = \frac{r^2}{2}(\theta - \sin\theta)$$

$\theta = 0$ Area $= 0$

$\theta = \pi$ Area $= \frac{\pi r^2}{2}$

Point on the unit circle

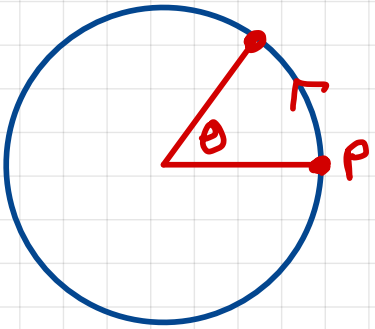


t in seconds

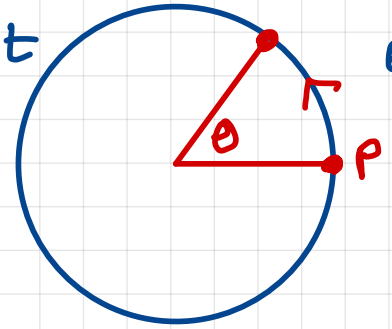
θ is in radians

$\theta(t)$ theta is a function of t

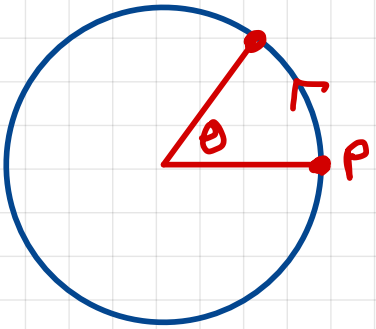
$\theta(0) = 0$



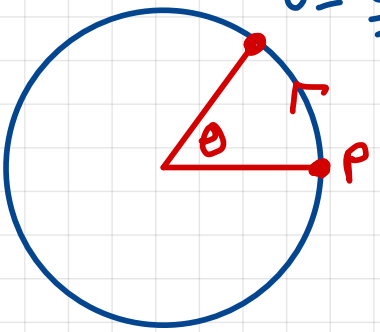
$\theta = t$



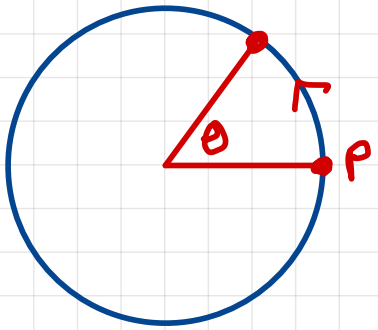
$\theta = 2t$



$\theta = 4t$



$\theta = \frac{t}{2}$



$\theta = \frac{t}{4}$

Time taken to complete on revolutions, winners and losers.

$f(\theta)$	Period T
t	2π
$2t$	π
$4t$	$\pi/2$
$t/2$	4π
$t/4$	8π
kt	$2\pi/k$

k is positive value

$\theta = kt$

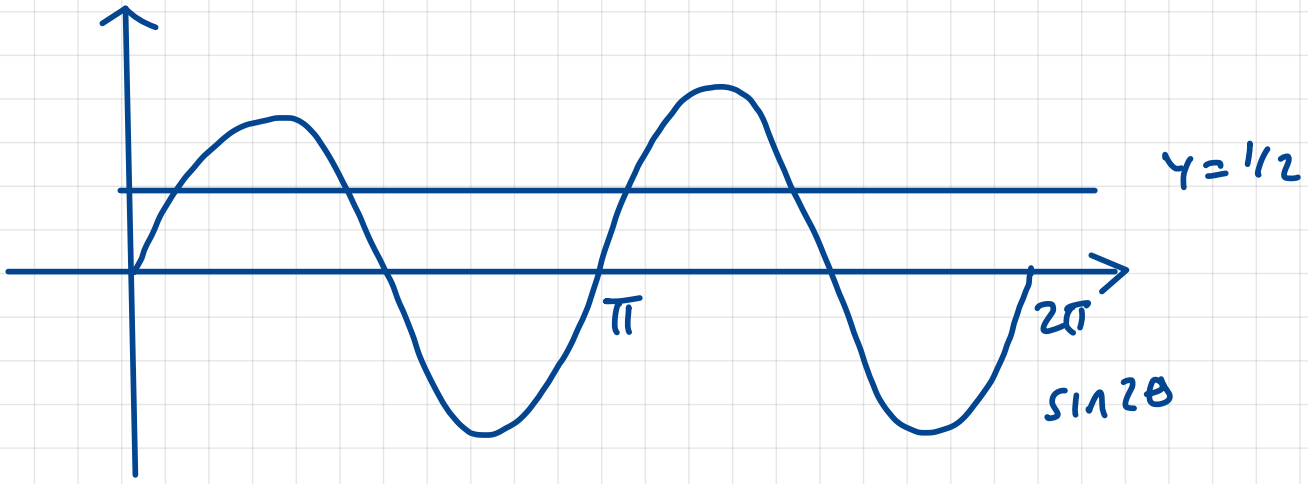
$\theta = 2\pi \quad t = \frac{2\pi}{k}$

$\sin k\theta \quad \cos k\theta \quad T = \frac{2\pi}{k}$

$\tan k\theta \quad T = \frac{\pi}{k}$

Example Solve

$\sin 2\theta = \frac{1}{2}$ $\quad 0 \le \theta \le 2\pi$



Solution

$2\theta = \sin^{-1}(1/2)$

n is a positive integer

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{6}+2\pi, \frac{5\pi}{6}+2\pi, \frac{\pi}{6}+2n\pi, \frac{5\pi}{6}+2n\pi$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \quad ---- \frac{\pi}{12}+n\pi, \frac{5\pi}{12}+n\pi$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}$